

Non-Probabilistic Decoherent Histories and Causal Histories in Everettian Quantum Mechanics

André L. G. Mandolesi

Departamento de Matemática, Universidade Federal da Bahia

Salvador-BA, Brazil

E-mail: `andre.mandolesi@ufba.br`

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Abstract

D. Wallace has combined the Decoherent Histories formalism and Everettian Quantum Mechanics, in an attempt to solve the preferred basis problem. This in turn serves as a foundation for his proof of the Born rule. But approximations used in decoherence depend on the probabilistic interpretation of Born weights, resulting in a circular reasoning. We contest his arguments that the approximations are valid even without probabilities, and show that, without them, the combination of those theories leads to unacceptable results. There is a proliferation of ill behaved histories, compromising his proof of the Born rule. We propose the development of a new causal histories formalism, which might improve the situation. Histories of small amplitude are discarded, not on probability grounds, but for suffering too much interference, which blurs causal relations. The remaining histories might still suffer small amounts of interference, leaving open the possibility of experimental verification.

1 Introduction

The Many Worlds Interpretation, or Everettian Quantum Mechanics (EQM) [EI57, DG73], is an attempt to solve the measurement problem of Quantum Mechanics. It rejects the Measurement Postulate, and applies the rest of the usual formalism to all systems, even macroscopic ones, at all times, including measurements. It concludes that a measurement is just entanglement of device and observer with the measured system, on a basis determined by their interaction. Device and observer evolve into a macroscopic superposition of different versions of themselves, each correlated to a state of the basis and registering only the corresponding result. The collapse of the wavefunction is illusory, due to the fact that, since evolution is linear and interference is negligible, each version of the observer is unaware of the other components of the superposition (which are called *branches* or *worlds*).

EQM faces a *preferred basis problem*: how to decompose a macroscopic quantum state into branches which behave approximately as the classical reality we observe. There is also a *probability problem*: evolution via Schrödinger's equation is deterministic, and all measurement results are obtained in different branches, so why do quantum experiments seem probabilistic, following the Born rule (which was eliminated with the Measurement Postulate)?

An idea to solve this last problem, proposed by Deutsch [Deu99] and developed by Wallace [Wal10, Wal12], uses decision theory to show that rational agents, following EQM, would decide

on bets about quantum experiments as if results were probabilistic and followed the Born rule. Such high level approach needs branches where devices and agents can exist as macroscopic quantum systems, so it requires a previous solution to the first problem. Wallace tries to obtain it via decoherent histories, but, without Born's rule, approximations used in decoherence, on the basis of negligible probabilities, are invalidated [Bak07]. So these solutions form a circular argument.

Wallace has proposed other ways to justify the approximations, which we argue are not valid. We show that, without them, combining decoherent histories and EQM leads to strange results. It opens a cornucopia of bizarre branches to emerge, lacking minimal conditions for use of decision theory. As a result, assumptions used in his proof of the Born rule become compromised, invalidating it.

As a possible way out, we propose the development of a new formalism of *causal histories*. In it, the Born weight of a branch is seen, at first, not as a probability, but as an indication of its resistance to interference. Branches of small amplitude are discarded not on probability grounds, but for suffering so much interference that it is impossible to track causal relations in their histories. If successful, such formalism would solve the preferred basis problem, providing branches similar to our classical reality. These could then be used in the decision theoretic proof of the Born rule.

The results we obtain may also point a way to settle EQM's *testability problem*: how to differentiate it experimentally from the usual Quantum Mechanics. If our conclusions are confirmed to be actual consequences of EQM, they would be in such disagreement with observation that EQM should be discarded as a failed theory. On the other hand, if the probability problem can be solved, our worst predictions can be disregarded for happening only on highly unlikely branches. But other branches can exhibit tiny deviations from usual quantum predictions, providing a way to test EQM, if our experimental capabilities ever become precise enough.

In section 2 we review the measurement problem of Quantum Mechanics, the Everettian interpretation and its problems, and the decoherent histories formalism.

We start section 3 rejecting Wallace's arguments that, even without probabilities, approximations used in decoherence remain valid. Then we analyze what happens when the decoherent histories formalism, stripped of its probabilistic elements, is combined with EQM. We also show that quasi-classical branches can be quite different from our classical reality, and it may be possible, in EQM, to have measurements or branch decompositions in almost orthogonal bases, with important consequences.

Section 4 questions the definition of history borrowed from the decoherent histories formalism. Without a Born rule allowing histories of small amplitude to be neglected, all of them must be considered, no matter how bizarre, and most exhibit a breakdown of causality at the macroscopic level. On the other hand, all macroscopic states exist at all times, suggesting a static macroscopic perspective. We suggest exploring the relation between Born weights, interference and causality to get a better definition of histories. This may lead to a new causal histories formalism, which might provide a better solution to the preferred basis problem.

Section 5 summarizes our conclusions. The problems found, we must stress, are not intended as predictions of observable results. Hopefully a Born rule can be proven, allowing them to be dismissed as unlikely. But in proving such rule they must be taken into account, and their effect on Wallace's proof is analyzed in [Man15].

2 Preliminaries

2.1 The measurement problem

In the Copenhagen interpretation of Quantum Mechanics (CQM), the Measurement Postulate states that, if a system is in a state

$$|\psi\rangle = \sum_i c_i |i\rangle, \quad (1)$$

with $\langle i | j \rangle = \delta_{ij}$ and $\sum |c_i|^2 = 1$, is measured with respect to the basis $\{|i\rangle\}$, the result will be one (and only one) of the i 's, and the state collapses to the corresponding $|i\rangle$. Also, results are probabilistic, according to the following rule.

Born Rule. *The probability of result i is $p_i = w_i$, where w_i is its Born weight,*

$$w_i = |c_i|^2 = |\langle i | \psi \rangle|^2. \quad (2)$$

This postulate agrees with experimental data, but is conceptually ambiguous. It sets measurements apart from other quantum processes, which obey the deterministic linear Schrödinger equation, but lacks a precise definition of what are measurements. These might be distinguished for involving a classical macroscopic system, like an observer, but if this system's particles obey Schrödinger's equation, how can they collectively produce a nonlinear probabilistic¹ process? And how does the collapse of the quantum state happen? Many attempts have been made to solve this *measurement problem*, such as hidden variables theories, Bohmian mechanics, nonlinear Schrödinger equations, and others, each with its own difficulties [WZ14, Aul00].

This relates to the problem of whether Quantum Mechanics remains valid as systems get bigger, with Classical Mechanics emerging from it. In the usual view, quantum superpositions should not happen at the macroscopic level, lest we observe Schrödinger cats. But nothing in the quantum formalism seems to induce their disappearance in large systems, quite to the contrary. So many physicists consider CQM valid only for microscopic systems, with another theory being needed to explain the quantum-classical transition. This point of view becomes problematic as quantum phenomena are verified at increasingly larger scales, or for research in fields like quantum cosmology. Some see decoherence as an explanation for the emergence of classicality, but it is questionable whether it eliminates superpositions, or merely wipes out interference between their components, which survive nonetheless.

2.2 Everettian quantum mechanics (EQM)

A solution, proposed by H. Everett III [EI57, DG73], rejects the Measurement Postulate, and applies the rest of the quantum formalism even to macroscopic systems. Evolution is always deterministic, even in measurements, following Schrödinger's equation. It leads to macroscopic superpositions, but also explains why observers do not perceive them. If not for some unsolved problems, it might explain quantum measurements, and provide the missing link between quantum and classical mechanics.

In EQM, a measurement is just entanglement of the measuring device with whatever is being measured. More precisely, a *measuring device* for a basis $\{|i\rangle\}$ of a system is any apparatus, in a quantum state $|D\rangle$, which interacts in such a way that, if the system is in state $|i\rangle$, the composite state evolves as²

$$|i\rangle \otimes |D\rangle \longmapsto |i\rangle \otimes |D_i\rangle, \quad (3)$$

¹Probabilistic even in principle, not simply due to lack of knowledge about the states of the particles.

²For simplicity, we assume the system remains in state $|i\rangle$, but this is not necessary.

where $|D_i\rangle$ is a new state of the device, registering result i . Linearity of Schrödinger's equation implies that, if the system is in state (1), the composite state evolves as

$$|\psi\rangle \otimes |D\rangle = \left(\sum_i c_i |i\rangle \right) \otimes |D\rangle \mapsto \sum_i c_i |i\rangle \otimes |D_i\rangle. \quad (4)$$

This final state is to be accepted as an actual quantum superposition of macroscopic states. But it will not be perceived as such by an observer looking at the device, as, by the same argument, his state $|O\rangle$ will evolve into a superposition, according to

$$\left(\sum_i c_i |i\rangle \otimes |D_i\rangle \right) \otimes |O\rangle \mapsto \sum_i c_i |i\rangle \otimes |D_i\rangle \otimes |O_i\rangle,$$

with $|O_i\rangle$ representing a state in which he saw result i . By linearity, each component $|i\rangle \otimes |D_i\rangle \otimes |O_i\rangle$ evolves independently, as if the others did not exist, as long as interference is negligible. In section 2.3 we show why this assumption may be reasonable.

Everett's interpretation of this final state is that the observer has split into different versions of himself, each seeing a distinct result. Each version evolves as if the initial state had been $|i\rangle \otimes |D\rangle \otimes |O\rangle$, so he does not feel the splitting, nor the existence of his other versions. Each component is called a *world* or a *branch*, and this evolution of one world into a superposition of many is called a *branching process*. So in EQM all possible results of a measurement actually happen, but in different worlds. The observer in state $|O_i\rangle$ only thinks the system has collapsed into $|i\rangle$ because he does not see the whole picture, with all other results and versions of himself.

Problems that plague the Copenhagen Interpretation disappear in EQM, but new ones come along, as discussed below.

2.2.1 Probability problem

In EQM, any result i with $c_i \neq 0$ is obtained with certainty when measuring (1), even if only one version of the observer sees it. The probability problem is how to reconcile this with the experimental record, which indicates that results are probabilistic and follow the Born rule.

It has a qualitative aspect, of how probabilities can emerge from a deterministic theory. In classical mechanics, processes can appear random due to our ignorance of details, but in EQM one must explain randomness even if the quantum state and its evolution are perfectly known. There is also the quantitative aspect of accounting for probability values. By a *Born-like rule* we mean any result explaining why, in an Everettian universe (i.e. one governed by EQM), quantum experiments would appear probabilistic, with probabilities given by the Born weights (2). Many attempts have been made to obtain such result [EI57, Gra73, AL88, Han03, Zur05, BHZ06].

Deutsch [Deu99] proposed an adaptation of decision theory [PI09, Kar14] to EQM, to show that, in an Everettian universe, it would be rational to make decisions, related to bets on the results of quantum experiments, as if the outcomes were probabilistic, with Born weights playing the role of probabilities. The idea was further developed by Wallace [Wal10, Wal12], which presented a formal proof. But use of decision theory requires worlds where narratives with agents, measurements and payoffs make sense. This involves solving first the preferred basis problem, described next.

2.2.2 Preferred basis problem

Decomposition of a state like (4) may not be unique [Zur81], so it is not clear whether there is one basis which gives the correct description in terms of actual worlds, or how to find it.

Also, Everett's description of quantum measurements assumes that EQM gives rise, at least in some branches, to complex macroscopic structures, like measuring devices and observers, that behave classically, most of the time, up to a good approximation. So the preferred basis problem consists in finding a natural way to decompose the quantum state of a macroscopic system into branches which behave like the classical reality we observe (even if not all of them, and not all the time).

Wallace [Wal12] has proposed adapting the decoherent histories formalism, described in section 2.3, to solve this problem. But, as we discuss in section 3, the probability problem may invalidate such solution.

2.2.3 Testability problem

Solving these problems would give EQM a better theoretical status than CQM, and might shed light in the question of whether experiments can tell these theories apart. Many physicists disregard EQM as, in their view, it is not testable, making the same predictions as CQM. But if that is the case, every quantum experiment ever performed is a test of EQM as much as it was of CQM. Impossibility of testing which better describes our universe would not make EQM worse than CQM. Had EQM been developed first, CQM might be the one disregarded for not making new predictions.

Granted, one may feel uneasy with a theory predicting undetectable other worlds. But any theory, even classical mechanics, has elements which can not be directly observed, but are accepted since other consequences have been confirmed.

Anyway, there is no guarantee that EQM and CQM are experimentally equivalent. Everett's description of measurements assumes unitary quantum mechanics gives rise to devices and observers, branches do not interfere, and the Born rule holds somehow. Validity of these assumptions depends on solving the previous problems, and answers we have so far indicate EQM might not precisely replicate CQM's predictions. In a worst case scenario, the attempted solution to the preferred basis problem may lead to predictions at odds with observations, as shown in section 3. In the best case, if the probability problem is solved, such bad predictions can be dismissed as unlikely, and perhaps EQM can be tested, either through small deviations from the Born rule, or some new macroscopic quantum phenomena. There have been proposals of how to do it [Deu86, Pla97, Pag99], but they are beyond our present experimental capability.

2.3 Histories formalisms

Wallace's approach [Wal12] to the preferred basis problem is based on the decoherent histories formalism, which we review below.

2.3.1 Decoherence

Decoherence [Zur02, Sch07, JZK⁺03] is a process by which an open quantum system loses some quantum characteristics, as it interacts and gets entangled with its environment.

Many models show some states are more robust with respect to this interaction, having a stronger tendency to remain disentangled. We say such *pointer states* are *selected* by the environment, which measures them in the sense of (3). In some cases they give an orthogonal basis, but in others they form an overcomplete set of vectors, as in the quantum brownian motion, where they are minimum-uncertainty Gaussian packets (coherent states) $|q, p\rangle$. This is considered a good paradigm for a macroscopic system, as through the scattering of particles the environment constantly measures its position and momentum (with some uncertainty). This helps explain why such systems are observed in states of fairly well defined position and momentum.

As the environment is differently affected by distinct pointer states, and such differences spread across its many degrees of freedom, it rapidly evolves into (almost) orthogonal states. Off-diagonal elements (*coherences*) of the reduced density matrix of the system, with respect to the pointer states, decay extremely fast. This (almost) eliminates interference between such states, and we say the system has *decohered*. Close pointer states take longer to decohere, but this problem can be reduced by coarse graining, as we discuss in section 2.3.3. As systems get bigger, it is difficult to shield them from the environment, and decoherence becomes ever present.

As the reduced density matrix becomes diagonal, it formally resembles a classical probabilistic mixture. This is seen as an important step in the quantum-classical transition, but should not be interpreted as if the system was now in a classical state. Tracing out the environment hid information about how the system is entangled to it, but the composite system remains in a pure quantum state. Adepts of CQM must explain the disappearance of all, but one, components of the mixture.

In EQM, all components remain, but entangled to (almost) orthogonal states of the environment. In this view, decoherence is just entanglement at work, a measurement of pointer states by the environment, which selects a basis for decomposition into branches, and (almost) eliminates interference between them. What distinguishes it is that it happens continuously, and, for all practical purposes, is irreversible, as information spreads through the environment's many degrees of freedom.

The reduced density matrix of an open system follows a master equation, and its dynamics lacks unitarity, which is central to Wallace's arguments. So in describing the evolution of branches he turns to the consistent/decoherent histories formalisms, which apply to closed quantum systems.

2.3.2 Consistent Histories (CH)

The *consistent histories* formalism, conceived by Griffiths [Gri84, Gri02] and further developed by Omnès [Omn88, Omn99], aims to describe quantum processes in ways which allow the use of classical (Boolean) logic and classical probabilities. It identifies conditions allowing us to assign classical probabilities to sets of alternative histories, conceived as sequences of events or propositions about a system.

Its point of view is opposite to the Everettian one: quantum evolution is always stochastic, and quantum states are only tools for calculating probabilities in a set of possible evolutions, only one of which happens. Still, parts of the formalism adapt well to the Everettian setting, if properly reinterpreted. We present a simplified version³, assuming a normalized pure initial state ψ_0 at time t_0 .

A *quantum sample space* is an orthogonal projective decomposition of Hilbert space, a family $\{P_\alpha\}$ of orthogonal projection operators with $\sum_\alpha P_\alpha = \mathbb{1}$ and $P_\alpha P_\beta = \delta_{\alpha\beta} P_\alpha$. It represents an exhaustive set of mutually exclusive events or propositions about the system. The Heisenberg picture is used, with $P_\alpha(t) = U(t, t_0)^{-1} P_\alpha U(t, t_0)$, where $U(t, t_0)$ is the unitary time evolution operator given by Schrodinger's equation.

A *history space* is a sequence $\{P_{\alpha_1}(t_1)\}, \dots, \{P_{\alpha_n}(t_n)\}$ of quantum sample spaces, at times $t_0 < t_1 < \dots < t_n$. A *history* $\alpha = (\alpha_1, \dots, \alpha_k, \dots, \alpha_n)$ is a sequence of events, specifying one $P_{\alpha_k}(t_k)$ from each sample space. In each history space only one history happens. Even if α happens, we can not say that, at time t_k , the system is in a state in the image of $P_{\alpha_k}(t_k)$, for the ontology of CH is based on histories, not quantum states. States are only artifacts of the

³Reflecting its stochastic point of view, CH is usually presented using density operators. Sometimes it includes a final state, or allows quantum sample spaces to be branch dependent.

formalism, and a different history space might yield another “true” history, whose projector at t_k could even be orthogonal to $P_{\alpha_k}(t_k)$.

To a history α we associate a *branch state vector* $\psi_\alpha = P_{\alpha_n}(t_n) \cdots P_{\alpha_1}(t_1)\psi_0$. In CQM, with sample spaces representing projective measurements, it is (up to normalization and time translation) the final state in case of sequence α of results, which has probability $p_\alpha = \|\psi_\alpha\|^2$. In EQM, it is one of the branches resulting from the evolution of ψ_0 , and we are yet to make sense of probabilities. In CH, it has no such interpretations, being only a tool to obtain the probability of history α happening, which, under appropriate conditions, is postulated to be, once more,

$$p_\alpha = \|\psi_\alpha\|^2 = \|P_{\alpha_n}(t_n) \cdots P_{\alpha_1}(t_1)\psi_0\|^2. \quad (5)$$

A history space $\{\bar{P}_{\bar{\alpha}_k}(t_k)\}$ is a *coarse-graining* or *coarsening* of $\{P_{\alpha_k}(t_k)\}$ if each $\bar{P}_{\bar{\alpha}_k}(t_k)$ is a sum of $P_{\alpha_k}(t_k)$ ’s. Conversely, $\{P_{\alpha_k}(t_k)\}$ is a *fine-graining* or *refinement* of $\bar{P}_{\bar{\alpha}_k}(t_k)$. This fits in the formalism by bundling history spaces into a *history algebra*, specified by taking, at each t_k , a *quantum event algebra*, which is a Boolean algebra⁴ of orthogonal projectors. The Boolean condition implies projectors commute, so their ranges are mutually orthogonal if the intersection is $\{0\}$.

Given a coarsening $\{\bar{P}_{\bar{\alpha}_k}(t_k)\}$ of $\{P_{\alpha_k}(t_k)\}$, each coarser history $\bar{\alpha}$ can be seen as a family of finer ones. We write $\alpha \in \bar{\alpha}$ if, for each k , the range of $\bar{P}_{\bar{\alpha}_k}(t_k)$ contains that of $P_{\alpha_k}(t_k)$, so $\psi_{\bar{\alpha}} = \sum_{\alpha \in \bar{\alpha}} \psi_\alpha$. CH requires the p_α ’s to behave as classical probabilities, so they must be additive, $p_{\bar{\alpha}} = \sum_{\alpha \in \bar{\alpha}} p_\alpha$. Hence there must be no interference between histories, and $\langle \psi_\alpha | \psi_\beta \rangle = 0$ ⁵ if $\alpha \neq \beta$. History spaces satisfying this condition are called *consistent*⁶ (relative to ψ_0), and are the only ones allowed in CH.

A consistent history space is seen as a valid, or allowed, set of questions about the system, at different times. In such space, only one sequence of answers, or history α , turns out to be true, with probability p_α . CH dismisses many quantum paradoxes by noting they involve inconsistent history spaces, so the problem lies in us asking an invalid set of questions. Of course, it is debatable whether it really solves the paradoxes, or just forbids us asking inconvenient sets of questions.

Two consistent history spaces can be *incompatible*, in the sense that they can not be combined into a single consistent one. Either one can be used to describe a quantum process, but not both at once. This is the source of much criticism. If in each space one history actually happens, are there many equally real but incompatible histories? Is it possible to impose some condition that all “true” histories be compatible in some sense, even if their history spaces are not? Can the formalism be supplemented with new conditions to single out one consistent set?

A history space has a *branching structure* (relative to ψ_0) if histories do not merge after diverging, i.e. if α and β satisfy $\alpha_i \neq \beta_i$ and $\alpha_j = \beta_j$, for some $i < j$, one of them has zero probability. Consistency and branching are related as follows [Gri93, Wal12].

Branching-Consistency Theorem.⁷ *Any history space with a branching structure is consistent, and the converse holds for some consistent refinement of it.*

2.3.3 Decoherent Histories (DH)

Classicality is more than its probability laws, and CH admits histories that are far from classical. The *decoherent histories* formalism, developed by Gell-Mann and Hartle [GMH90, GMH93], combines CH and decoherence to get special history spaces with a more classical behavior. It

⁴With operations $P_1 \wedge P_2 = P_1 P_2$, $P_1 \vee P_2 = P_1 + P_2 - P_1 P_2$, and $\neg P = \mathbb{1} - P$.

⁵The condition $\text{Re}\langle \psi_\alpha | \psi_\beta \rangle = 0$ suffices for additivity, but is problematic for composite systems [Dio04].

⁶The terminology varies. Wallace uses consistency for additivity of probabilities, and decoherence for non-interference. The concept of decoherence from section 2.3.1 is related but not equivalent to this one.

⁷Branching-Decoherence Theorem, in Wallace’s terminology.

seeks not only (approximate) consistency⁸, but also *quasi-classicality*, meaning histories approximately follow classical equations of motion, interrupted at times (as in quantum measurements) by some quantum behavior.

In DH, the space of relevant variables of the subsystem of interest is partitioned into cells Σ_α , large in comparison with the coherence length (below which decoherence is not effective), yet small enough for the required precision. This determines a quantum sample space given by operators $P_\alpha \otimes \mathbb{1}_{\text{env}}$, where

$$P_\alpha = \int_{\Sigma_\alpha} dx |x\rangle\langle x|, \quad (6)$$

and $\mathbb{1}_{\text{env}}$ is the identity operator for the environment's Hilbert space (with the environment including the irrelevant variables of the subsystem). A history α specifies, at a sequence of times t_k , in which Σ_{α_k} the subsystem is⁹. Coarsenings and refinements are obtained by varying the cell size.

While in CH consistency is a condition to admit a history space, in DH it arises via decoherence. For large enough cells, states in the range of different P_α 's are distinct enough to quickly get entangled to (almost) orthogonal states of the environment¹⁰. Orthogonality tends to subsist, as the environment evolves, for its many degrees of freedom keep *records* of the history. For example, particles scattered by the system in different directions, at distinct Σ_α 's, tend to remain in (almost) orthogonal states, and affect other degrees of freedom in distinct ways. Erasing from the environment all traces of the history is impossible in practice. Of course, as different states of the environment evolve, they can spontaneously develop similar components, blurring their records, but their Born weights would be negligibly small.

As records ensure (almost) consistency, (almost) additive probabilities can be assigned to histories, as in CH. Some interference between histories subsists, but it is negligible if they are coarse enough. Deviations in the additivity of probabilities should be irrelevant, as long as they are too small to be detected experimentally.

The Branching-Consistency Theorem gives an (approximate) branching structure for a refinement. But it will not be in terms of projectors $P_\alpha \otimes \mathbb{1}_{\text{env}}$ into smaller cells $\Sigma_{\tilde{\alpha}}$, as it will require projecting environmental states onto the different records.

Models show that, if histories are coarse enough, and the system has enough inertia to resist noise from the environment, histories with non-negligible probabilities will (approximately) follow classical equations of motion, with a stochastic force.

So a (almost) consistent history space seems to emerge naturally in DH, and its non-negligible histories are quasi-classical. However, we can still have incompatible descriptions of the evolution, as shown by Dowker and Kent [DK96].

3 The preferred basis problem without probabilities

Wallace [Wal12] uses DH to solve the preferred basis problem. But as EQM is not stochastic, and all histories happen, probabilities (5) should not be postulated, but obtained solving the probability problem. As his solution of this other problem depends on branches provided by the first solution, it forms a circular argument [Bak07].

He has argued that DH can be used without probabilities, giving other justifications for its approximations, instead of the usual negligibility of tiny probabilities. We do not agree with his arguments, and examine the consequences of adapting DH, stripped of its probabilistic

⁸In their terminology, medium decoherence.

⁹If necessary, different families of cells can be used at each time.

¹⁰This fails near cell boundaries, but if cells are not too large, distinguishing adjacent ones is irrelevant.

elements, to EQM. We show the resulting branches are too ill behaved to be of use in his proof of the Born rule.

3.1 Branch discontinuities

The Born rule allows us to ignore small amplitude components, corresponding to unlikely results. So tiny perturbations in a state do not alter much its physical meaning. Until the probability problem is solved, we can not assume it to be valid in EQM.

Let $|\psi_\epsilon\rangle = \sqrt{1-\epsilon}|0\rangle + \sqrt{\epsilon}|1\rangle$, where $|0\rangle$ and $|1\rangle$ represent macroscopically distinct states. Without a Born-like rule, or some other justification, we can not simply neglect $|1\rangle$ for small ϵ . So we must accept that the physical content of $|\psi_\epsilon\rangle$ might change drastically when ϵ goes from 0 (a single branch) to nonzero (two branches, both equally relevant). Hence arbitrarily small perturbations in a state can create lots of branches, none negligible. We call this phenomenon a *branch discontinuity*.

One can argue that no reasonable physical theory can have discontinuities like these, but the truth is we do not know EQM to be a good theory. Without a definitive explanation of how it connects with experiments, we must consider that it might lead to really bad predictions. So objections on such basis are not valid.

Wallace argues that, even without a probabilistic interpretation, the Hilbert space metric can measure approximations, i.e. states that are close in the metric have similar physical meanings. He presents several reasons [Wal12, p.253]:

- The metric tells us “when some emergent structure really is robustly present”, and when it “goes away when we slightly perturb the microphysics”.

But this is precisely the problem: if tiny perturbations can create new branches, branch decomposition is not a robust structure of the formalism.

- “Hilbert space norm is a perfectly objective feature of the physics, prior to any considerations of probability”.

Experimentally, we measure probabilities, which in CQM are connected to the norm. In EQM, the metric appears only in the requirement of unitary evolution, but without a probabilistic interpretation there is no reason to require norm conservation. So unitarity, norm and metric might be mathematical artifacts carried over from CQM, with no actual role in EQM.

- The norm “is just a natural measure of state perturbations in Hilbert space, and that naturalness follows from considerations of the microphysical dynamics”.

Examples given indicate he refers to the fact that small changes in a state remain small as it evolves. But if dynamical conservation is what makes a norm or metric “natural”, are spatial or phase space metrics unnatural for classical dynamics?

- There is no “profound difference between the role of the Hilbert space metric in quantum physics and, say, the spatial metric in classical physics”.

Even if that is so, the spatial metric does not have the property he wants: close states in the spatial metric can differ much in velocity. Let us consider instead the phase space metric. We know close points represent physically similar states because classically there is a clear connection between theory and experiment. We understand what phase space points represent, how to measure their properties, and that these are continuous in the metric: close points give similar results. In EQM, the usual link between quantum formalism and experiments has been severed. We are trying to rebuild it, but, until one manages to confirm the experimental meaning of a macroscopic state $|\psi\rangle$ in EQM, there is no guarantee that close states, in the Hilbert metric, generate similar observations.

Note that, no matter how far apart two wave packets get, their states remain at an almost constant distance in Hilbert space. Which is smaller than the distance between the physically equivalent states $|\psi\rangle$ and $-|\psi\rangle$. If this metric is such a lousy measure of how different two states are, why, without a Born-like rule, should we expect it to be a good measure of similarity?

Also, were we to accept the idea that small branches can be neglected, even without a probabilistic interpretation, there might be better ways than decision theory to obtain the Born rule, including Everett's original proof [EI57] or those of [Gra73, Han03, BHZ06].

3.2 Non-Probabilistic Decoherent Histories (NPDH)

By *Non-Probabilistic Decoherent Histories* we mean the parts of DH which do not depend on probabilities, reinterpreted from an Everettian perspective. In it, branch discontinuities come into play, usual approximations in DH are no longer valid, and all those “approximately” and “almost” permeating section 2.3.3 become relevant. In a slight abuse of terminology, we call (5) Born weights.

The range of (6) consists of wavefunctions with support in Σ_α . As evolution via Schrödinger's equation does not preserve compact supports, they instantly develop tails spreading across all other cells. In CQM this is not strange, as such tail gives a negligible probability that the system will jump to another cell. In EQM, it means new branches, corresponding to all other cells, pop into existence. Without a Born-like rule, they must be considered as real and relevant as any.

In NPDH, lots of highly non-classical histories, which were negligible in DH due to their tiny Born weights, become relevant. Essentially all histories happen, some ill behaved even by quantum standards, with lots of macroscopic quantum jumps. Even from quasi-classical branches there is a continuous sprouting of tiny weird sub-histories. The “penalty” for such bad behavior is a drastic reduction in Born weight, but, without a Born-like rule, this may be irrelevant.

One may argue that macroscopic quantum jumps can not happen, as they violate conservation laws. But these apply to averages over the whole state, not branchwise. Of course, if EQM does allow large violations in branches, and no Born-like rule is found to discard them as unlikely, it should be considered an invalid theory.

This compromises the very idea of measurement. A device designed to work as in (3) will always malfunction in some branches, and spill out all sorts of random results. Of course, this is not an acceptable description of our Universe, unless it is complemented by a Born-like rule, allowing us to neglect most possibilities. But, until such rule is obtained, this is the picture we have to deal with in NPDH.

It can be elucidating to compare this with Feynman's formalism of path integrals. One considers at first all possible paths, no matter how weird. Destructive interference clears the picture, allowing us to consider only the more well behaved ones. But the more complicated ones are not really eliminated, they just have incredibly small Born weights. Which, without a Born-like rule, does not mean they can be neglected.

Decoherence does not really happen in terms of cells with precise boundaries, as pointer states $|\pi_x\rangle$ tend to be like gaussian packets, peaked at a point x , but with small tails across all space. It does not make much difference in DH, as the tails give negligible probabilities, but in NPDH it might be better to redefine (6) as

$$P_\alpha = \int_{\Sigma_\alpha} dx |\pi_x\rangle \langle \pi_x|. \quad (7)$$

Evolution of the $|\pi_x\rangle$'s can be more stable, but now the P_α 's are only almost orthogonal to each other. A state in the range of P_α can have a tiny component in the range of another $P_{\alpha'}$.

As it decoheres, there will be a component in which the environment records α , but also a tiny one, albeit equally relevant, with a record of α' .

Evolution of the environment also comprises all possibilities, including tiny components where its state changes drastically, and records disappear or get replaced by wrong ones. As records are not reliable, tiny branches can suffer interference from larger ones. For this, the difference in Born weights must be of a great many orders of magnitude, even more so if the branches and their histories are very different.

3.3 Quasi-classicality and Dissimilarity

In DH, non negligible histories behave quasi-classically. In NPDH, all histories are relevant, most exhibit strange behaviors, but some will be quasi-classical. Were we to consider only such branches, for example in an anthropic argument, could we be sure they are similar to the macroscopic world we know?

In CQM, the probabilistic interpretation of Born weights is used not only in direct quantum measurements, but also to explain decay rates of atoms, chemical reactions, etc. Quasi-classicality is no substitute for this. A branch can approximately follow classical equations without having stable atoms or macroscopic objects. For example, a branch in which all atoms have disintegrated, and all particles behave, at the macroscopic level, like a classical gas, fits the definition of quasi-classical. It will have an incredibly small Born weight, but, without a Born-like rule, is as relevant as any. In NPDH, the only reason to assume the existence of branches similar to our world is that everything will happen in some branch. But these will not be typical in any sense, not even among quasi-classical ones. They may have larger Born weights, but without a Born-like rule we need another reason for this to be relevant.

This presents a big obstacle for decision theoretic proofs of a Born-like rule, which depend on the existence of high level structures (agents, experiments, bets, etc.), and on their behavior being somewhat similar to what we are used to. Any such proof should take into account the following condition:

Dissimilarity. *Even quasi-classical branches can not be assumed to be similar to our macroscopic reality. So no assumptions can be justified, and no possibilities excluded, based on our physical experience.*

For example, branches in which a broken glass spontaneously becomes whole again should happen in NPDH, and some may even be quasi-classical, so Dissimilarity encompasses even the Second Law of Thermodynamics.

3.4 Almost Orthogonality

Orthogonality acquires its role in CQM via Measurement Postulate, which states that measurements¹¹ are in the eigenvector bases of Hermitian operators, and uses orthogonal projections to define Born weights. Orthogonal states are mutually exclusive, in the sense that they can be eigenvectors for different values of an observable, and measuring one eigenvector never results in the other values. In CH, orthogonality appears in the projectors used to define histories, and in the requirement of consistency. In DH it is present in the projectors (6), which are an idealization of the more realistic, almost orthogonal, operators (7). Almost consistency is achieved by entanglement with almost orthogonal states of the environment.

In EQM, we must question if emergent structures such as branches will turn out orthogonal to each other. We can not impose it hoping EQM replicates conditions from CQM (like measurements in orthogonal bases), nor as a necessity to obtain probabilities. In particular, we

¹¹We refer only to projective measurements, as more general ones, like POVMs, can be reduced to them.

must ask why observables correspond to Hermitian operators, or why histories should be defined using orthogonal projectors. Of course, a Hermitian Hamiltonian is necessary for unitary evolution, but it is not clear how this translates into a decomposition of the quantum state into orthogonal branches.

A measurement in EQM is a normal quantum process, which happens to lead to different versions of measuring device and observer, entangled to distinct states of the measured system, and evolving independently. Linearity of Schrödinger's equation seems to allow this even for a nonorthogonal basis. For example, let $|0\rangle$ and $|1\rangle$ be orthonormal states of a system, and $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. It is not immediate why, in EQM, it is not possible for a device D to measure in the basis $\{|+\rangle, |1\rangle\}$, so that

$$|+\rangle \otimes |D\rangle \mapsto |+\rangle \otimes |D_+\rangle, \quad \text{and} \quad |1\rangle \otimes |D\rangle \mapsto |1\rangle \otimes |D_1\rangle. \quad (8)$$

By Dissimilarity, one can not argue no known device can do this, or that our knowledge (based on CQM) forbids it. Nor can one say that a measurement of $|0\rangle$ by D ,

$$|0\rangle \otimes |D\rangle = (\sqrt{2}|+\rangle - |1\rangle) \otimes |D\rangle \mapsto \sqrt{2}|+\rangle \otimes |D_+\rangle - |1\rangle \otimes |D_1\rangle,$$

results in a probability greater than 1 for $|+\rangle$, as this requires a Born-like rule. A good argument is that, if $|D_+\rangle$ and $|D_1\rangle$ are macroscopically distinct, differing in the states of lots of particles, they must be almost orthogonal, so (8) violates unitarity. But it would not apply if D measured an almost orthogonal basis, instead of $\{|+\rangle, |1\rangle\}$.

So it seems EQM might allow almost orthogonal measurements, or, more generally, branchings along almost orthogonal subspaces. Use of NPDH reinforces this, since emergence of branches is more natural in terms of the almost orthogonal projectors (7), selected by decoherence with the environment.

In practical terms, is almost orthogonality so different from perfect orthogonality? Once we have a Born-like rule, the answer might be no, as slight deviations from orthogonality would cause negligible changes in probabilities. Until then, it is yes, for a couple of reasons:

1. The idea of mutual exclusivity is lost. If there can be a measuring device for an almost orthogonal basis $\{|\epsilon\rangle, |1\rangle\}$, where $|\epsilon\rangle = \sqrt{1-\epsilon}|0\rangle + \sqrt{\epsilon}|1\rangle$ for small $\epsilon > 0$, then a measurement of $|0\rangle$ by it results $|1\rangle$ in some branches.
2. Any set of mutually orthogonal states is linearly independent, but this is not true with almost orthogonality, if the number of states is large enough. In principle, a linear combination of a huge number of almost orthogonal, macroscopically distinct states, might result in a state macroscopically quite different from all of them¹². So a quantum state might admit two decompositions in terms of quite different sets of branches. Another way to look at this is that the set of branches which can emerge from EQM includes incompatible decoherent history spaces.

4 Interference and Macroscopic Causality

Some of the problems of NPDH may stem from the definition of histories, which works well for DH, but seems too generic when we no longer have probabilities to prune the worst branches. We propose tightening the definition by requiring a sense of causality between events in a history, which may be quantified in terms of how little interference is present. An adequate formalism based on this idea is still in development, so we lay out only the main notions and possible ways to develop them.

¹²One can build wavefunctions concentrated in a region, whose sum concentrates far away. The example is mathematically artificial, but shows the need to explain why it can not happen in realistic cases.

4.1 All Histories or No History?

Usually, the Born weight of a quantum state concentrates in some cells, but it has, at any time, components in all of them, as the wavefunction is precisely zero in a whole one only in exceptional cases. In DH, the probabilistic interpretation allows us to neglect most cells. In a visual analogy, imagine cells in shades of gray, those of larger Born weight being darker. As dark spots move, and maybe divide into lighter ones, we get a picture of branching histories, each as probable as its final darkness level. All those dark enough to be discernible should be quasi-classical.

In NPDH, all nuance disappears, and cells are either black (nonzero component) or white (absolutely no component). Barred exceptional cases, they are black all the time. In this case, is it reasonable to say all histories happen, or there is no history? Sticking to a visual analogy, is a black paper full of black paths, or has it no paths?

Formally, we can define histories or paths as we wish. If paths are defined as arbitrary sequences of black dots, a black paper is full of paths. But we usually hope such labels reflect our intuitive ideas, which matter in choosing axioms and interpreting results. For paths, we might want a color different from the paper, and maybe some notion of continuity to connect the dots. In transposing the definition of histories from DH to NPDH, we must question whether it retains its intuitive interpretation.

In the definition of history α , consecutive projectors $P_{\alpha_i}(t_i)$ and $P_{\alpha_{i+1}}(t_{i+1})$ are linked by $U(t_{i+1}, t_i)$, and a component in $\Sigma_{\alpha_{i+1}}$ only counts if it came from Σ_{α_i} . But, in NPDH, a state in Σ_{α} , at t_i , generates components in all $\Sigma_{\alpha'}$'s at t_{i+1} . Conversely, each $\Sigma_{\alpha'}$ will have a superposition of components coming from all Σ_{α} 's. Even if $U(t_{i+1}, t_i)$ shows α_i contributes a component to α_{i+1} , we can not say α_{i+1} happens at t_{i+1} because of α_i at t_i . Other cells would cause α_{i+1} anyway, and the effect of α_i may even be destructive, reducing the Born weight (if it matters) of α_{i+1} via interference.

Causal relations get lost, if everything happens, all the time, in some branch, and interference prevents tracking, in a meaningful way, the consequences of anything. Consider for example the following events:

- A. There is a glass at the edge of the kitchen table;
- B. All glasses are stored inside the kitchen cabinet;
- C. There is no glass in the kitchen;
- D. There is glass shattered on the floor beside the kitchen table.

An observer, presented with D, may think it follows from A. But in NPDH it may just as well result from B, with the glass breaking after tunneling from the cabinet, or even C, with atoms spontaneously joining to form pieces of glass on the floor. In fact, two points of view might be valid in NPDH:

- All histories: there is glass on the floor for these reasons, and all others, at once, as any macroscopic state is generated by (and generates) all others.
- No history: there is glass on the floor because, at all times, there will always be glass on the floor, in some branch.

The first one results from using the definition of histories from DH. That it may seem unreasonable is an indication that such definition may be too broad for NPDH. The second one abandons that definition, and adopts the intuitive idea that histories should involve change, with events ceasing to exist and new ones coming about.

4.2 Causal Histories

It may be possible to avoid such extremes and reach a more reasonable concept of history, based on a non-probabilistic interpretation of Born weights.

Suppose, at time t_1 , the Born weight is equally divided among events A, B and C, and, moments later, at a time t_2 , there is a reasonable weight on D. It is not correct to say D happens at t_2 because of any specific event at t_1 , but we can attribute its Born weight mostly to A, since B and C generate very small components on D, which interfere negligibly with the larger one coming from A.

Imagine, on the other hand, that at t_1 most Born weight is on B or C, and A has a very tiny component. In this case the contribution to D coming from B and C may be comparable to that of A. Interference between these histories keeps us from identifying any one of them as the source of the weight on D.

So Born weights admit a non probabilistic interpretation, measuring resistance to interference. This gains importance as it allows the establishment of causal relations for larger branches, less affected by interference. It brings shades of gray back to our picture: wave packets of large Born weight, suffering little interference, move along darker quasi-classical paths, amidst an ocean of almost white cells, whose lightness flickers, in indiscernible ways, as they suffer interference from everywhere. Such cells remain in a state of perpetual but timeless existence: they are present at all times, but with their Born weights, and their specific microstates, fluctuating meaninglessly, with no causal relations connecting them to other cells in a significant macroscopic narrative. A cell will only be part of an evolving narrative once a discernible wave packet passes through it, linking it causally in time with other cells.

This may lead to a formalism of *causal histories*, including only those with Born weight large enough to resist interference and sustain causal relations. Histories of low Born weight are discarded not on probability grounds, but for lack of causality: sequences of unrelated events are not histories in any meaningful sense of the word, only artifacts of a definition that was too broad.

The amplitude below which histories can be discarded should not be absolute. As they keep on branching, eventually even the largest one will fall below any fixed value. As its Born weight becomes much smaller than the total weight of the rest, one might think it would suffer significant interference. But interference between cells decays rapidly with their distance, so only the weight of close ones should matter. A tiny wave packet can evolve causally as long as there are no larger ones in its vicinity.

We say a history $\alpha = (\alpha_1, \dots, \alpha_n)$ is *causal* if it suffers little interference. Making this precise requires an appropriate interference measure. We present some possibilities, which compare the effect on Σ_{α_n} of all other histories with that of α :

$$\begin{aligned} I_1(\alpha) &= \sum_{\substack{\tilde{\alpha} \neq \alpha, \\ \tilde{\alpha}_n = \alpha_n}} \frac{\|\psi_{\tilde{\alpha}}\|}{\|\psi_{\alpha}\|}, & I_3(\alpha) &= \sum_{\substack{\tilde{\alpha} \neq \alpha, \\ \tilde{\alpha}_n = \alpha_n}} \frac{|\langle \psi_{\tilde{\alpha}} | \psi_{\alpha} \rangle|}{\|\psi_{\alpha}\|^2}, \\ I_2(\alpha) &= \frac{\|P_{\alpha_n}(t_n)\psi_0 - \psi_{\alpha}\|}{\|\psi_{\alpha}\|}, & I_4(\alpha) &= \frac{|\langle P_{\alpha_n}(t_n)\psi_0 - \psi_{\alpha} | \psi_{\alpha} \rangle|}{\|\psi_{\alpha}\|^2}. \end{aligned}$$

For a chosen interference function, α will be causal if $I(\alpha) \ll 1$. Note that $I_1 \geq I_3$, $I_2 \geq I_4$, and, as $P_{\alpha_n}(t_n)\psi_0 = \sum_{\tilde{\alpha}_n = \alpha_n} \psi_{\tilde{\alpha}}$, we have $I_1 \geq I_2$ and $I_3 \geq I_4$. If $I_3(\alpha) \ll 1$ (resp. $I_4(\alpha) \ll 1$), a suitable refinement gives $I_1(\alpha) \ll 1$ (resp. $I_2(\alpha) \ll 1$). If the P_{α} 's are mutually orthogonal, as in (6), the condition $\tilde{\alpha}_n = \alpha_n$ can be removed in I_3 .

I_1 and I_3 neglect interference between other histories, allowing lots of tiny $\psi_{\tilde{\alpha}}$'s, which might nearly cancel out, to be counted as if they were accumulating and having a large effect on ψ_{α} . I_2 and I_4 take it into account, which can also cause problems. Let $\tilde{\alpha}$ and $\hat{\alpha}$ be histories,

distinct from α , with $\psi_{\tilde{\alpha}} \cong \psi_{\alpha}$ and $\psi_{\tilde{\alpha}} \cong -\psi_{\alpha}$. Should we say that $\psi_{\tilde{\alpha}}$ and $\psi_{\tilde{\alpha}}$ cancel out and the branch on Σ_{α_n} is caused by α , or that ψ_{α} and $\psi_{\tilde{\alpha}}$ cancel out and $\tilde{\alpha}$ causes Σ_{α_n} ? Are both α and $\tilde{\alpha}$ causal, or neither one?

I_1 and I_2 take into account all components generated on Σ_{α_n} , while I_3 and I_4 ignore those orthogonal to ψ_{α} . This last approach may be preferable since if, for example, ψ_{α} and $\psi_{\tilde{\alpha}}$ differ only on the records stored in the environment, the presence of $\psi_{\tilde{\alpha}}$ at Σ_{α_n} does not keep us from recognizing ψ_{α} as caused by α . In such case, a finer graining (involving the environmental variables) can separate the two histories into finer ones which are causal in the sense of I_1 and I_2 .

For I_1 , causal histories have an approximate branching structure. The same is not true of I_2 , as can be seen in the situation described above, or the others.

Environmental records should make $\langle \psi_{\tilde{\alpha}} | \psi_{\alpha} \rangle$ approach 0 extremely fast, as $\tilde{\alpha}$ and α become more different. For I_3 or I_4 , this means for α not to be causal there must be an $\tilde{\alpha}$ for which $\|\psi_{\tilde{\alpha}}\| \gg \|\psi_{\alpha}\|$, with their ratio being of a great many orders of magnitude, and increasing rapidly the more different the histories are. In other words, α should be causal if its Born weight is not too small, or there are no close histories of much larger weight (nor farther ones with an extremely big weight ratio).

An appropriate threshold for the inequality $I(\alpha) \ll 1$, above which histories are to be discarded as non causal, may be necessary. It must not be too arbitrary, and its precise value must not affect results significantly. Or, perhaps, it might be better to work with a gradation of causality, instead of a binary alternative.

For any of the proposed measures, it is possible for a history to suffer lots of interference at some steps, which latter dissipates. A stronger causality condition might require negligible interference at each step of a history.

If an adequate causality condition can be established, and the formalism works as expected, it would show Born weights can play a natural role in EQM, prior to a Born-like rule. This can be an important step towards proving such rule, which, once obtained, might justify the choice of threshold, as one which allows probabilities of causal histories to fluctuate, due to interference, only within the desired precision. As such histories can still suffer tiny amounts of interference, it opens the possibility of experimental observation. However, it should require a precision well beyond our present capabilities.

We note that a similar idea has been proposed by Hanson [Han03], with a different interpretation of the effect of interference on small worlds: they would be “mangled”, with observers ceasing to exist or remembering events from larger worlds. This seems like a very literal reading of what interference means, and implicitly assumes some sort of narrative makes sense for such worlds. But in any case his conclusion is the same as ours: branches whose amplitude is small enough can be neglected. Bunih et al. [BHZ06] also try to eliminate small branches, by assuming Hilbert space is discrete.

5 Conclusion

Wallace’s proposed solution to the preferred basis problem is promising, but incomplete. His attempt to justify approximations used in decoherence on non-probabilistic grounds is not satisfactory. So they still depend on the probabilistic interpretation of Born weights, and require solving first the probability problem. But his solution of that problem, via decision theory, relies on the solution of the preferred basis problem to provide appropriate branches, creating a circular argument.

As we have shown, lack of probabilities, or other reasons, to justify the approximations is not some technical nuisance with little effect on the result. Instead, the combination of decoherent

histories and Everettian quantum mechanics, without a probabilistic interpretation, would lead to strange consequences:

1. Absent a non-probabilistic reason to disregard them, small branches are as relevant as large ones. Branch decomposition is not a robust feature of the formalism, due to branch discontinuities: arbitrarily small changes in a quantum state can create lots of non negligible branches. Approximations used in decoherence can no longer be justified.
2. All macroscopic histories happen and are equally relevant, no matter how erratic. Branches can exhibit macroscopic quantum jumps all the time, with no macroscopic sense of causality. Those allowing a meaningful macroscopic narrative are the exception, and even they keep sprouting bizarre subbranches. Environmental records are unreliable, and tiny branches can suffer significant interference from much larger ones. An alternative view is that, since all macroscopic states are present at all times, there is no real history.
3. Large scale behavior governed, most of the time, by classical equations is not enough to make quasi-classical branches resemble our world. The microscopic world must provide appropriate building blocs, like atoms and molecules, but the stability and behavior of these rely on the Born rule. Disintegration of all atoms, or other phenomena usually disregarded as unlikely, cause a huge reduction in Born weight, so large branches do not have such problems. But without a Born-like rule there is no reason to prefer them, unless one cherry picks those similar enough to our world to give the desired results.
4. Orthogonality might not mean in EQM the same as in the Copenhagen version. Measurements (i.e. branch decompositions) in almost orthogonal bases may be possible, and orthogonal states might not be mutually exclusive upon measurement. An almost orthogonal set of branches can be linearly dependent, so, even allowing for coarsenings or refinements, branch decompositions might not be unique. Two decompositions of the same state can involve quite different sets of branches.

Of course, if EQM really leads to such results, it should be discarded as a bad model for physical reality. But if a Born-like rule can be obtained, they can be dismissed for affecting only highly unlikely branches. However, if a proof of such rule depends on properties of the branch decomposition, these problems must be taken into consideration, to avoid a circular argument. In particular, the consequences for Wallace’s proof are analyzed in [Man15].

A solution could be the development of the proposed causal histories formalism, giving Born weights a non-probabilistic role, as measures of causality or resistance to interference. Histories of small amplitude are disregarded not for being unlikely, but for suffering so much interference that it becomes impossible to track causal relations between its events.

Such formalism is in its early stages of development, and many questions need answer to ensure it would work as expected. First of all, we must determine the most appropriate way to measure interference, and how much of it is too much for causality. Maybe there is not a unique answer to this question, and different approaches might be better suited for each situation.

If everything works as we anticipate, causal histories should give us well behaved branches. They would not only be quasi-classical, but should also have all ingredients needed to form our classical reality: stable atoms, molecules working as usual, the different states of matter, etc. Basically, all phenomena usually considered likely due to their large Born weights should be present, while the weirdness associated with low weights should be mostly absent.

Such branches might provide an appropriate framework for proving a Born-like rule, via decision theory or some other approach. In particular, use of causal histories instead of decoherent ones might solve some of the problems found in Wallace’s proof. If the other problems can also be solved, we would have a non circular explanation of how Born weights can work as probabilities in Everettian Quantum Mechanics.

References

- [AL88] D. Albert and B. Loewer, *Interpreting the many worlds interpretation*, Synthese **77** (1988), no. 2, 195–213.
- [Aul00] G. Auletta, *Foundations and interpretation of quantum mechanics: In the light of a critical-historical analysis of the problems and of a synthesis of the results*, World Scientific Publ., 2000.
- [Bak07] D. Baker, *Measurement outcomes and probability in everettian quantum mechanics*, Stud. Hist. Philos. Sci. Part B **38** (2007), no. 1, 153–169.
- [BHZ06] R.V. Buniy, S.D.H. Hsu, and A. Zee, *Discreteness and the origin of probability in quantum mechanics*, Phys. Lett. B **640** (2006), no. 4, 219 – 223.
- [Deu86] D. Deutsch, *Three connections between everett’s interpretation and experiment*, Quantum Concepts in Space and Time (R. Penrose and C.J. Isham, eds.), New York; Oxford University Press, 1986, pp. 215–225.
- [Deu99] D. Deutsch, *Quantum theory of probability and decisions*, Proc. Roy. Soc. Lond. **455** (1999), 3129–3137.
- [DG73] B. DeWitt and N. Graham (eds.), *The many-worlds interpretation of quantum mechanics*, Princeton University Press, 1973.
- [Dio04] L. Diosi, *Anomalies of weakened decoherence criteria for quantum histories*, Phys. Rev. Lett. **92** (2004), 170401.
- [DK96] F. Dowker and A. Kent, *On the consistent histories approach to quantum mechanics*, J. Stat. Phys. **82** (1996), no. 5, 1575–1646.
- [EI57] H. Everett III, *Relative state formulation of quantum mechanics*, Rev. Mod. Phys. **29** (1957), no. 3, 454–462.
- [GMH90] M. Gell-Mann and J.B. Hartle, *Quantum mechanics in the light of quantum cosmology*, Complexity, entropy and the physics of information (W.H. Zurek, ed.), Addison-Wesley, Reading, 1990, pp. 425–458.
- [GMH93] ———, *Classical equations for quantum systems*, Phys. Rev. D **47** (1993), 3345–3382.
- [Gra73] N. Graham, *The measurement of relative frequency*, The Many Worlds Interpretation of Quantum Mechanics (B. DeWitt and N. Graham, eds.), Princeton University Press, 1973, pp. 229–253.
- [Gri84] R.B. Griffiths, *Consistent histories and the interpretation of quantum mechanics*, J. Stat. Phys. **36** (1984), no. 1, 219–272.
- [Gri93] ———, *Consistent interpretation of quantum mechanics using quantum trajectories*, Phys. Rev. Lett. **70** (1993), 2201–2204.
- [Gri02] ———, *Consistent quantum theory*, Cambridge University Press, 2002.
- [Han03] R. Hanson, *When worlds collide: Quantum probability from observer selection?*, Found. Phys. **33** (2003), no. 7, 1129–1150.
- [JZK⁺03] E. Joos, H.D. Zeh, C. Kiefer, D.J.W. Giulini, J. Kupsch, and I.O. Stamatescu, *Decoherence and the appearance of a classical world in quantum theory*, Springer, 2003.
- [Kar14] E. Karni, *Axiomatic foundations of expected utility and subjective probability*, Handbook of the Economics of Risk and Uncertainty (M. Machina and K. Viscusi, eds.), vol. 1, North-Holland, 2014, pp. 1 – 39.

- [Man15] A. Mandolesi, *Analysis of Wallace’s decision theoretic proof of the Born rule in Everettian quantum mechanics*, [arXiv:quant-ph/1504.05259](#) (2015).
- [Omn88] R. Omnès, *Logical reformulation of quantum mechanics. I. Foundations*, J. Stat. Phys. **53** (1988), no. 3, 893–932.
- [Omn99] ———, *Understanding quantum mechanics*, Princeton University Press, 1999.
- [Pag99] D.N. Page, *Can quantum cosmology give observational consequences of many-worlds quantum theory?*, AIP Conference Proceedings **493** (1999), no. 1, 225–232.
- [PI09] G. Parmigiani and L. Y. T. Inoue, *Decision theory: principles and approaches*, John Wiley & Sons, Ltd, Chichester and UK, 2009.
- [Pla97] R. Plaga, *On a possibility to find experimental evidence for the many-worlds interpretation of quantum mechanics*, Found. Phys. **27** (1997), no. 4, 559–577.
- [Sch07] M.A. Schlosshauer, *Decoherence: and the quantum-to-classical transition*, The Frontiers Collection, Springer, 2007.
- [Wal10] D. Wallace, *How to prove the born rule*, Many Worlds? Everett, Quantum Theory & Reality (S. Saunders, J. Barrett, A. Kent, and D. Wallace, eds.), Oxford University Press, 2010, pp. 227–263.
- [Wal12] ———, *The emergent multiverse: Quantum theory according to the everett interpretation*, Oxford University Press, 2012.
- [WZ14] J.A. Wheeler and W.H. Zurek, *Quantum theory and measurement*, Princeton Series in Physics, Princeton University Press, 2014.
- [Zur81] W.H. Zurek, *Pointer basis of quantum apparatus: Into what mixture does the wave packet collapse?*, Phys. Rev. D **24** (1981), no. 6, 1516.
- [Zur02] W. Zurek, *Decoherence and the transition from quantum to classical - revisited*, Los Alamos Science **27** (2002), 86–109.
- [Zur05] W.H. Zurek, *Probabilities from entanglement, born’s rule $p_k = |\psi_k|^2$ from envariance*, Phys. Rev. A **71** (2005), 052105.